Nonlinear Feedback Deployment and Retrieval of Tethered Satellite Systems

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The Lyapunov approach for controller design is applied to the tethered subsatellite deployment and retrieval problem. The tether dynamic model includes tether mass as well as in-plane and out-of-plane librations. The nonlinear feedback tension control law developed guarantees stability of the closed-loop system and is used in combination with out-of-plane thrusting during retrieval. Simulation results show the tension and the combined tension and thruster control laws perform well for deployment and retrieval, respectively, even when aerodynamic drag is considered.

Nomenclature

C_{d_t}, C_{d_p}	= drag coefficients for the tether and subsatellite, respectively
d_t	= diameter of the tether
$\boldsymbol{F}$	= external force vector
$\boldsymbol{F}_G$	= gravity gradient force vector
F_y	= external force in the y direction
h, h_0	= vertical component of the deployed tether length and reference altitude, respectively
L	= maximum tether length
$\boldsymbol{M}$	= external torque vector
$\boldsymbol{M}_G$	= gravity gradient torque vector
M_x, M_z	= external torques about the x and z axes
m_p	= subsatellite mass
$\boldsymbol{R}_0$, $\boldsymbol{R}_p$, $\boldsymbol{R}_t$	= vectors from the center of the Earth to the spacecraft, m_p, and an element of the tether, respectively
T	= tether tension at the spacecraft attach point
$\boldsymbol{y}$	= vector from the spacecraft attach point to the subsatellite ($\lvert \boldsymbol{y} \rvert = y$)
$\boldsymbol{\xi}$	= vector from the spacecraft attach point to a mass element on the tether ($\lvert \boldsymbol{\xi} \rvert = \xi$)
ρ_t	= mass per unit length of the tether
ρ_0	= reference density of the atmosphere
σ	= Earth's velocity of rotation

Introduction

PROPOSAL of tethered subsatellite systems for several important applications has led to many investigations dealing with their dynamics and control. A comprehensive survey of these investigations is given by Misra and Modi.[1] Previous investigations in this area have shown that the nonlinear dynamics of the tethered satellite system are far from simple. Deployment of a tethered satellite is such a stable process that open-loop exponential-linear-exponential control schemes[2] may be used. However, the motion of the system during retrieval is basically an unstable process due to excitation of both in-plane and out-of-plane librations and insufficient tension during the terminal phases. Open-loop control laws similar to those used for deployment will often result in disastrous behavior.

Many feedback control laws have been proposed for the deployment and retrieval of tethered satellites, among which is the pioneering work by Rupp,[3] whose control law is based on modulating the tension of the tether as a function of the commanded and actual lengths. This type of control law was applied to a more sophisticated model, which included out-of-plane motion, by Baker et al.[4] Bainum and Kumar[5] applied linear optimal control theory to devise a tether tension control law based on feedback of tether length, length rate, in-plane pitch angle, and its rate. A control law that includes additional nonlinear feedback of the out-of-plane tether angular rate was proposed by Modi et al.[2] Recently, another nonlinear feedback tension control law was introduced by Liangdong and Bainum[6] that includes nonlinear in-plane and out-of-plane angles and angular rates. Although these control laws have been shown to be effective for various conditions through simulations, the design methodology does not appear to be based on stability considerations of the nonlinear system. Also, all of the previously mentioned control laws require a feed-forward length command that must be chosen carefully for proper operation of the control laws.

The Lyapunov (mission function) approach was first implemented for the design of a deployment/retrieval control law by Fujii and Ishijima.[7] They presented a highly nonlinear tension control law that can be used for both deployment and retrieval and does not require feed-forward length commands. However, the model used was rather simple in that tether mass and flexibility, out-of-plane motion, and aerodynamic effects were neglected. Two other types of control laws derived via the Lyapunov approach were introduced by Vadali[8] using the same model as Fujii and Ishijima. One is an alternative tension control law and the other a velocity control law that allows fast retrieval by attempting to maintain a constant pitch angle.

A three-dimensional treatment of this problem, including out-of-plane dynamics but neglecting tether mass and aerodynamics, is provided by Vadali and Kim.[9] Two control laws are derived in Ref. 9. The first is based on partial decomposition of the equations of motion and utilization of a two-dimensional control law developed in Ref. 8. The other is based on a Lyapunov function that takes into consideration out-of-plane motion. It is shown in Ref. 9 that the control laws are effective when used in conjunction with out-of-plane thrusting.

In this study, another nonlinear feedback tension control law is derived using the Lyapunov approach for a model that includes tether mass and out-of-plane motion. The tension control law is used in combination with out-of-plane thrusters to improve convergence in the terminal phases of retrieval.

Presented as Paper 90-3460 at the AIAA Guidance, Navigation, and Control Conference, Portland, OR, Aug. 20–22, 1990; received Sept. 7, 1990; revision received Jan. 8, 1991; accepted for publication Jan. 21, 1991.

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Analysis of the stability of the closed-loop system is also given, and the effectiveness of the tension control law in the presence of aerodynamic drag is verified via simulation.

System Equations of Motion

As mentioned previously, the dynamics of the tethered satellite system is complex in its general form when we take into account terms such as flexibility of the tether, translation of the system center of mass, and off-centered attachment of the tether. In this study, these effects are neglected for simplicity based on the assumption that they have little effect on the general trend of the motion of the system. We further simplify the dynamics by making the following assumptions.

1) The mass of the spacecraft is assumed to be sufficiently larger than the mass of the subsatellite so that deviation from the reference circular orbit due to deployment/retrieval may be neglected.

2) The tether is assumed to be uniform and straight.

3) The subsatellite is assumed to be a small spherical body.

The reference coordinate systems are shown in Fig. 1. In reality, the massive tether will be curved due to aerodynamic effects. It will also vibrate laterally. For this case, the y axis should be defined to be aligned with the straight line connecting the spacecraft attach point and the subsatellite attach point, and the deviation of the tether from this line will add to the error of the simplified model used herein. With this definition of the y axis, the in-plane and out-of-plane angles can be computed by measuring the relative position of one of the attach points with respect to the other. Some possible schemes of accomplishing this may be through the use of telemetry systems on the spacecraft and the subsatellite, or laser equipment.

The equations of motion are derived by the Newton-Euler method according to the procedure outlined in Ref. 10 and the assumptions mentioned above as the following:

$$M + M_G = \frac{\mathrm{d}}{\mathrm{d}t}(y \times m_p\dot{y}) + y \times m_p\ddot{R}_0 + \frac{\mathrm{d}}{\mathrm{d}t}\int_0^y \xi \times \dot{\xi}\rho_t \,\mathrm{d}\xi + \frac{1}{2}\rho_t y(y \times \ddot{R}_0) \tag{1a}$$

$$F + F_G + T = m_p\ddot{R}_p + \int_0^y \ddot{R}_t\rho_t \,\mathrm{d}\xi \tag{1b}$$

It should be noted here that, in formulating Eqs. (1), the Netwon-Euler method is applied to a moving control volume. This control volume is defined to include the subsatellite and the portion of the tether from a point attached to the tether (and therefore, having deployment velocity $\dot{l}$) at the tether-spacecraft attach point, to the subsatellite attach point. It is necessary to define the control volume in this manner to correctly reflect the dynamics of the tether being deployed by a reel system. If the control volume is not defined as above, but defined from a point attached to the spacecraft, the force required to accelerate a mass particle initially in the spacecraft to the deployment velocity will appear in the tension equation. Consequently, time derivatives of integral functions should be evaluated with the lower limit of integration initially replaced by y_0 with $\dot{y}_0 = \dot{l}$, and later setting $y_0 \to 0$. Then, by making use of Leibnitz's rule, the following identity can be derived:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^y f(\xi,t)\,\mathrm{d}\xi = \frac{\mathrm{d}}{\mathrm{d}t}\int_{y_0}^y f(\xi,t)\,\mathrm{d}\xi\Bigg|_{y_0 \to 0} = \int_0^y \frac{\mathrm{d}}{\mathrm{d}t}f(\xi,t)\,\mathrm{d}\xi \tag{2}$$

It is understood that Eq. (2) holds for all equations appearing herein.

The transformation between the flight-path-oriented X, Y, Z coordinate system to the tether attached coordinate system is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} C_\phi & S_\phi C_\theta & S_\theta S_\phi \\ -S_\phi & C_\phi C_\theta & S_\theta C_\phi \\ 0 & -S_\theta & C_\theta \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{3}$$

where C_x and S_x denote $\cos x$ and $\sin x$, respectively. The relationship between the angular velocity w in the x, y, and z coordinate system and the time derivatives of θ and ϕ is

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} (\Omega + \dot{\theta})C_\phi \\ -(\Omega + \dot{\theta})S_\phi \\ \dot{\phi} \end{pmatrix} \tag{4}$$

We obtain the following equations of motion after including the contributions due to the gravity gradient and performing the necessary mathematical operations:

$$\begin{aligned} M_x = {} & \left(m_p + \frac{\rho_t y}{3}\right)y^2 C_\phi \ddot{\theta} + 2\left(m_p + \frac{\rho_t y}{2}\right)y\dot{y}(\Omega + \dot{\theta})C_\phi \\ & - 2\left(m_p + \frac{\rho_t y}{3}\right)y^2\dot{\phi}(\Omega + \dot{\theta})S_\phi \\ & + 3\Omega^2\left(m_p + \frac{\rho_t y}{3}\right)y^2 S_\theta C_\theta C_\phi \end{aligned} \tag{5a}$$

$$\begin{aligned} M_z = {} & \left(m_p + \frac{\rho_t y}{3}\right)y^2\ddot{\phi} + 2\left(m_p + \frac{\rho_t y}{2}\right)y\dot{y}\dot{\phi} \\ & + \left(m_p + \frac{\rho_t y}{3}\right)y^2(\Omega + \dot{\theta})^2 C_\phi S_\phi \\ & + 3\Omega^2\left(m_p + \frac{\rho_t y}{3}\right)y^2 C_\theta^2 S_\phi C_\phi \end{aligned} \tag{5b}$$

$$\begin{aligned} F_y - T = {} & (m_p + \rho_t y)\ddot{y} - \left(m_p + \frac{\rho_t y}{2}\right)y(\Omega + \dot{\theta})^2 C_\phi^2 \\ & - \left(m_p + \frac{\rho_t y}{2}\right)y\dot{\phi}^2 + \Omega^2\left(m_p + \frac{\rho_t y}{2}\right)(y - 3yC_\theta^2 C_\phi^2) \end{aligned} \tag{5c}$$

where T is the tension of the tether at the attach point in the negative y direction. The external torques and forces are assumed to be due to aerodynamic drag and subsatellite thrusters.

The equations of motion may be nondimensionalized by defining the following variables:

$$\lambda = \frac{y}{L}, \quad \hat{T} = \frac{T}{m_p\Omega^2 L}, \quad \hat{F}_y = \frac{F_y}{m_p\Omega^2 L}$$

$$\hat{M}_{(\,)} = \frac{M_{(\,)}}{m_p\Omega^2 L^2}, \quad \tau = \Omega t$$

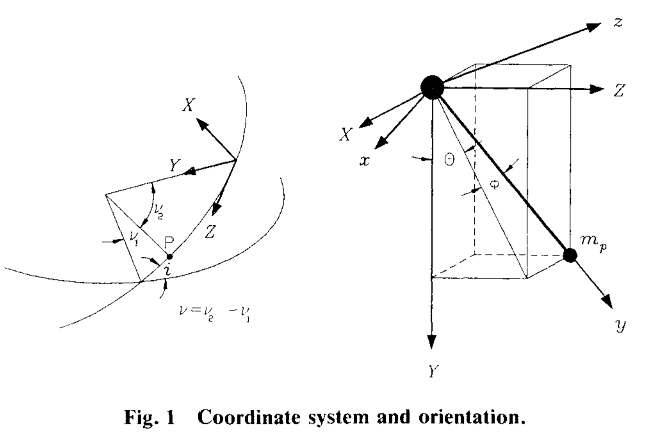

Fig. 1 Coordinate system and orientation.

Then, we may write the equations of motion in nondimensional form as

$$\hat{M}_x = \left(1 + \frac{c\lambda}{3}\right)\lambda^2 C_\phi \theta'' + 2\left(1 + \frac{c\lambda}{2}\right)\lambda\lambda'(1 + \theta')C_\phi$$
$$- 2\left(1 + \frac{c\lambda}{3}\right)\lambda^2 \phi'(1 + \theta')S_\phi + 3\left(1 + \frac{c\lambda}{3}\right)\lambda^2 S_\theta C_\theta C_\phi \quad (6a)$$

$$\hat{M}_z = \left(1 + \frac{c\lambda}{3}\right)\lambda^2 \phi'' + 2\left(1 + \frac{c\lambda}{2}\right)\lambda\lambda'\phi'$$
$$+ \left(1 + \frac{c\lambda}{3}\right)\lambda^2(1 + \theta')^2 S_\phi C_\phi + 3\left(1 + \frac{c\lambda}{3}\right)\lambda^2 C_\theta^2 S_\phi C_\phi \quad (6b)$$

$$\hat{F}_y - \hat{T} = (1 + c\lambda)\lambda'' - \left(1 + \frac{c\lambda}{2}\right)\lambda(1 + \theta')^2 C_\phi^2$$
$$- \left(1 + \frac{c\lambda}{2}\right)\lambda\phi'^2 + \left(1 + \frac{c\lambda}{2}\right)\lambda(1 - 3C_\theta^2 C_\phi^2) \quad (6c)$$

where $c = \rho_t L / m_p$ and $(\ )'$ denotes differentiation with respect to τ.

Aerodynamic Drag

One of the important proposed applications of the tethered subsatellite system is to conduct research in the layer of higher atmosphere, not accessible to either atmospheric vehicles or satellites, by deploying a research satellite from the Space Shuttle. Significant aerodynamic forces acting on the tether and subsatellite must be accounted for in such applications. The aerodynamic force vector acting on an element ΔL of the tether can be approximated by[11]

$$\Delta F = -\tfrac{1}{2} C_{d_t} \rho \Delta L d_t (V_x^2 + V_z^2)^{1/2} V \quad (7)$$

The atmospheric density and velocity of the element with respect to the atmosphere are defined by

$$\rho = \rho_0 e^{(h/h_0)}$$

$$\boldsymbol{V} = V_x \boldsymbol{i} + V_y \boldsymbol{j} + V_z \boldsymbol{k}$$
$$= [R_0 \sigma C_\nu S_i C_\phi - \dot{R}_0 C_\theta S_\phi - R_0(\dot{\theta} - \sigma C_i) S_\theta S_\phi]\boldsymbol{i}$$
$$+ [-R_0 \sigma C_\nu S_i S_\phi - \dot{R}_0 C_\theta C_\phi - R_0(\dot{\theta} - \sigma C_i) S_\theta C_\phi]\boldsymbol{j}$$
$$+ [-\dot{R}_0 S_\theta - R_0(\dot{\theta} - \sigma C_i) C_\theta]\boldsymbol{k}$$

The total aerodynamic torque acting on the system due to the tether can then be expressed by the components

$$M_{x_t} = -\tfrac{1}{2} C_{d_t} d_t (V_x^2 + V_z^2)^{1/2} V_z \rho_0 \int_0^y y e^{yC_\phi C_\theta / h_0}\, dy \quad (8a)$$

$$M_{z_t} = \tfrac{1}{2} C_{d_t} d_t (V_x^2 + V_z^2)^{1/2} V_x \rho_0 \int_0^y y e^{yC_\phi C_\theta / h_0}\, dy \quad (8b)$$

about the x and z axes. The y component of the aerodynamic force is

$$F_{y_t} = -\tfrac{1}{2} C_{d_t} d_t (V_x^2 + V_z^2)^{1/2} V_y \rho_0 \int_0^y e^{yC_\phi C_\theta / h_0}\, dy \quad (8c)$$

Since the subsatellite is assumed to be spherical, the aerodynamic torque and force components acting on the subsatellite are given by

$$M_{x_p} = -\tfrac{1}{2} C_{d_p} A_p |\boldsymbol{V}| \rho_0 e^{yC_\phi C_\theta / h_0} y V_z \quad (9a)$$

$$M_{z_p} = \tfrac{1}{2} C_{d_p} A_p |\boldsymbol{V}| \rho_0 e^{yC_\phi C_\theta / h_0} y V_x \quad (9b)$$

$$F_{y_p} = -\tfrac{1}{2} C_{d_p} A_p |\boldsymbol{V}| \rho_0 e^{yC_\phi C_\theta / h_0} V_y \quad (9c)$$

Out-of-Plane Thrusters

In this study, we assume that damping of the out-of-plane motion is provided by thrusters on the subsatellite whose thrust is a linear function of the time derivative of the out-of-plane angle. In real applications, the subsatellite must be controlled so that the thrusters are pointed in the correct direction if thrusters are to be used to control the out-of-plane motion. Even if the subsatellite is equipped with such a control system, pointing errors might cause thruster firings to excite the in-plane motion also. As an example, the attitude control system of the subsatellite to be used in the TSS-1 mission, scheduled to be launched in the near future, is designed to hold the yaw angle so that the out-of-plane thrusters are correctly oriented within an error bound of ± 3 deg.[12]

Assuming that the angular deviation in pointing direction is ψ about the y axis, the nondimensional torque generated by the thrusters about the x and z axes may be expressed as

$$\hat{M}_{\phi_x} = -K_\phi \lambda \phi' \sin\psi \quad (10a)$$

$$\hat{M}_{\phi_z} = K_\phi \lambda \phi' \cos\psi \quad (10b)$$

Now, the total external torques and forces due to aerodynamic drag and thrusters may be expressed in nondimensional form as

$$\hat{M}_x = \hat{M}_{x_p} + \hat{M}_{x_t} + \hat{M}_{\phi_x} \quad (11a)$$

$$\hat{M}_z = \hat{M}_{z_p} + \hat{M}_{z_t} + \hat{M}_{\phi_z} \quad (11b)$$

$$\hat{F}_y = \hat{F}_{y_p} + \hat{F}_{y_t} \quad (11c)$$

Controller Design by Lyapunov Method

The Lyapunov (mission) function method of designing feedback control laws for nonlinear systems has been applied to a variety of problems including spacecraft large angle maneuvers. A review of some of the past studies and explanation of the method can be found in Ref. 13. Basically, the method can be explained as follows.

Let V be a positive definite function of the states of a nonlinear dynamic system such that $V = 0$ if and only if the states reach the desired final state. If a control can be found as a function of the states such that the time derivative of the Lyapunov function V is globally negative definite, the control law guarantees V to converge to zero and the states to converge to the desired final state. Such a feedback control law is said to asymptotically stabilize the closed-loop system. If, however, $\dot{V}$ is negative semidefinite, it may be possible for the states to converge to other equilibrium configurations. Obviously, even if $\dot{V}$ is negative definite, the states may converge to values other than the desired final values if $V = 0$ for those values of the states. Nevertheless, we can guarantee that the desired final state is reachable in a local sense. Physically, this means that if the deployment/retrieval rate is not too large, the desired equilibrium point can be reached.

Although this method is attractive in that it leads to a feedback control law that stabilizes the nonlinear system, finding a suitable Lyapunov function is often very difficult. There is also no guarantee concerning performance of the control law, and in most cases, the feedback gains must be chosen by trial and error.

For the tethered satellite system, the desired final state after deployment or retrieval is

$$\lambda = \lambda_f, \qquad \dot{\phi} = \dot{\theta} = \dot{\lambda} = 0$$

The selection of a proper Lyapunov function is normally the most difficult step in the controller design procedure using the Lyapunov approach. There are no systematic procedures for constructing the Lyapunov function for all dynamic systems. Therefore, the Lyapunov function is usually constructed by skillfully selecting positive definite terms in the Lyapunov function such that when the Lyapunov function is differentiated, an expression for the control variable can be selected that will cause the derivative to become negative definite or negative semidefinite. Integrals of motion and energy expressions are often good starting points for selecting a Lyapunov function, although any positive definite function may be used theoretically. In the present study, the Hamiltonian is used to construct the Lyapunov function. The Hamiltonian for the model used here is

$$H = \frac{1}{2}\left\{\left(1 + \int_0^\lambda c\,\mathrm{d}\eta\right)\left(\lambda'^2 - 3\frac{R_0^2}{L^2}\right) + \left(\lambda^2 + \int_0^\lambda c\eta^2\,\mathrm{d}\eta\right)\right.$$
$$\left.\times\,(\phi'^2 + \theta'^2 C_\phi^2 + 3S_\theta^2 C_\phi^2 + 4S_\phi^2 - 3)\right\} \tag{12}$$

It can be verified that the Hamiltonian of the system is a constant of motion when tether length is assumed constant. The similarity of Eq. (12) with the constant of motion found in Ref. 9 should be noted. By utilizing the positive definite terms of H, a Lyapunov stable feedback control law can be derived for the tethered satellite system as follows.

Let us define V as follows:

$$V = \frac{1}{2}\left\{\left(1 + \int_0^\lambda c\,\mathrm{d}\eta\right)\lambda'^2 + K_1(\lambda - \lambda_f)^2\right.$$
$$+ \left(\lambda^2 + \int_0^\lambda c\eta^2\,\mathrm{d}\eta\right)(\phi'^2 + \theta'^2 C_\phi^2 + 3S_\theta^2 C_\phi^2 + 4S_\phi^2)$$
$$\left.+ K_2\left(1 + \frac{1}{3}\int_0^\lambda c\,\mathrm{d}\eta\right)(\phi'^2 + \theta'^2 C_\phi^2 + 3S_\theta^2 C_\phi^2 + 4S_\phi^2)\right\} \tag{13}$$

It can be verified easily that V is positive definite for $K_1 \geq 0$ and $K_2 \geq 0$ and is identically zero if the desired final state is reached. It is also apparent that, besides the desired final state, $V = 0$ for the following equilibrium states:

1) $\lambda = \lambda_f, \quad \lambda' = \theta' = \phi' = 0, \quad \theta = 0, \quad \phi = \pi$

2) $\lambda = \lambda_f, \quad \lambda' = \theta' = \phi' = 0, \quad \theta = \pi, \quad \phi = 0$

3) $\lambda = \lambda_f, \quad \lambda' = \theta' = \phi' = 0, \quad \theta = \pi, \quad \phi = \pi$

Note that equilibrium states 1 and 2 describe identical orientation of the tethered satellite system when the tether is deployed upward and the orientation described by 3 is equivalent to the desired final state. The control design process is initiated by neglecting the aerodynamic forces.

Differentiating Eq. (13) with respect to τ and using Eqs. (6) to eliminate the second derivatives, V' is obtained as

$$V' = \lambda'\left\{-\hat{T} + 3\left(1 + \frac{c\lambda}{2}\right)\lambda + K_1(\lambda - \lambda_f)\right.$$
$$\left.- \frac{2K_2}{\lambda}\left(1 + \frac{c\lambda}{2}\right)[\phi'^2 + (1 + \theta')\theta' C_\phi^2]\right\}$$
$$- K_\phi(K_2 + \lambda^2)\phi'^2 \tag{14}$$

It is clear that V' is rendered negative semidefinite for positive K_ϕ and K_3, by choosing

$$\hat{T} = 3\left(1 + \frac{c\lambda}{2}\right)\lambda + K_1(\lambda - \lambda_f) - \frac{2K_2}{\lambda}\left(1 + \frac{c\lambda}{2}\right)$$
$$\times\,[\phi'^2 + (1 + \theta')\theta' C_\phi^2] + K_3\lambda' \tag{15}$$

Then,

$$V' = -K_3\lambda'^2 - K_\phi(K_2 + \lambda^2)\phi'^2 \tag{16}$$

Since V' is negative semidefinite, equilibrium states other than the ones described by conditions 1, 2, and 3 may exist. Setting λ' and ϕ' to zero and substituting the control law of Eq. (15) into the equations of motion give the following nonlinear equations, which must be solved simultaneously for the other equilibrium states:

$$0 = (\theta'' + 3S_\theta C_\theta)C_\phi \tag{17a}$$

$$0 = [(1 + \theta')^2 + 3C_\theta^2]C_\phi S_\phi \tag{17b}$$

$$-K_1(\lambda - \lambda_f) + \frac{2K_2}{\lambda}\left(1 + \frac{c\lambda}{2}\right)(1 + \theta')\theta' C_\phi^2$$
$$= -\left(1 + \frac{c\lambda}{2}\right)\lambda(1 + \theta')^2 C_\phi^2 + \left(1 + \frac{c\lambda}{2}\right)\lambda(4 - 3C_\theta^2 C_\phi^2) \tag{17c}$$

Examination of Eqs. (17) shows that if $C_\phi S_\phi \neq 0$, Eq. (17b) cannot be satisfied. Therefore, the only possible values of ϕ are $\phi = 0, \pi/2, \pi, \ldots$. Further analysis of Eqs. (17) according to the procedure given in Ref. 13 yields the following results.

1) For $\phi = 0, \pi, \ldots$, equilibrium occurs when $\theta' = 0$ and $\theta = 0, \pi/2, \pi, \ldots$. The equilibrium tether length when $\theta = 0, \pi, \ldots$ is the desired final length and when $\theta = \pi/2, \ldots$, the equilibrium length satisfies,

$$K_1(\lambda - \lambda_f) + 3\left(1 + \frac{c}{2}\lambda\right)\lambda = 0$$

2) For $\phi = \pi/2, \ldots$, the tether length satisfies,

$$K_1(\lambda - \lambda_f) + 4\left(1 + \frac{c}{2}\lambda\right)\lambda = 0$$

and there are no restrictions on θ. Although this equilibrium state may seem to imply existence of a dynamic equilibrium state, this is not the case since, when $\phi = \pi/2, \ldots$, the tether aligned axis is aligned with the orbit normal X axis regardless of θ.

The equilibrium states described by 1 and 2 above can be shown to be unstable equilibrium states for the values of the gains chosen in the simulations to follow by examining the linearized models about those equilibrium states. Therefore, only two true equilibrium states exist—the desired final state and the reversed state ($\theta = \pi$, $\phi = 0$). By suitable choice of the control gains, the closed-loop system can be made to converge to the desired state.

Numerical Results

The tethered satellite is assumed to be deployed from the Space Shuttle Orbiter, which is in a 200-km circular orbit about the Earth. The values of constants common to all the simulations are given in Table 1. In practice, the values of the gains are selected by trial and error, so that performance is satisfactory and tension is always positive.

Table 1 System parameters

Description	Symbol	Value
Reference atmosphere density, kg/m^3	ρ_0	1.2321×10^{-13}
Atmosphere density scaling factor, m^{-1}	$(1/h_0)$	0.1513×10^{-3}
Tether drag coefficient	C_{d_t}	2.1
Subsatellite drag coefficient	C_{d_p}	2.2
Subsatellite mass, kg	m_p	150.0
Subsatellite cross section, m^2	ΔA_p	1.0
Tether mass density, g/m	ρ_t	1.5
Tether diameter, mm	d_t	1.0
Initial deployment velocity, km/s	$\dot{y}_0$	0.07
Angle of inclination, deg	i	90.0

Deployment

The deployment process is basically a stable operation, and the proposed nonlinear feedback control law is shown to be effective in Fig. 2. Deployment can be achieved within 1.5 orbits with very little oscillation in pitch and roll, even without damping of the out-of-plane motion via the thrusters, when aerodynamics is not considered. Although the initial in-plane angular deviation is large, it converges to zero quickly. The nondimensional tension is initially 0.65 (12.5 N) and settles to its equilibrium value of 86.4 N. When aerodynamic drag is considered, in-plane angular oscillations are more pronounced but die out to the equilibrium hangoff angle of approximately 24 deg. Also, initial tension is higher (57.6 N) due to the different set of gains used. The aerodynamic disturbance causes a small ($\approx 5\%$) steady-state error in the deployed length. The error may be eliminated either by increasing the commanded length or by implementing a stationkeeping control law at the end of the deployment process.

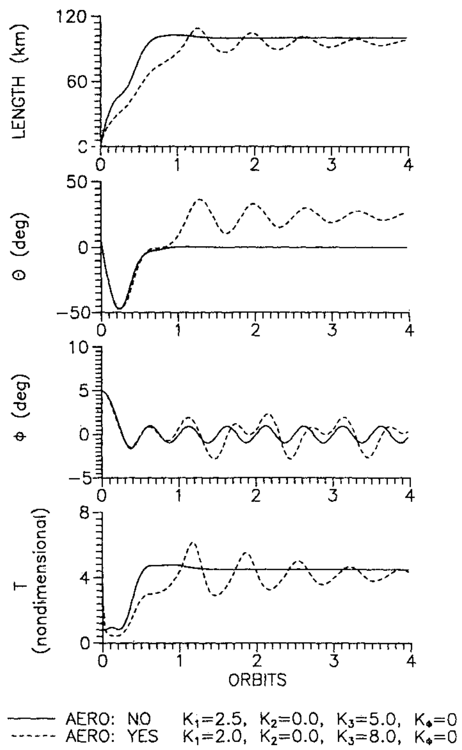

Fig. 2 Deployment ($\theta_0 = \phi_0 = 5.0$ deg).

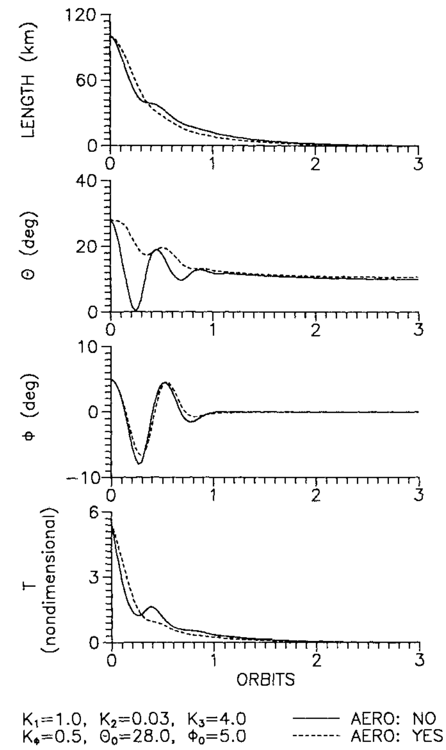

Fig. 4 Retrieval (with thrusters).

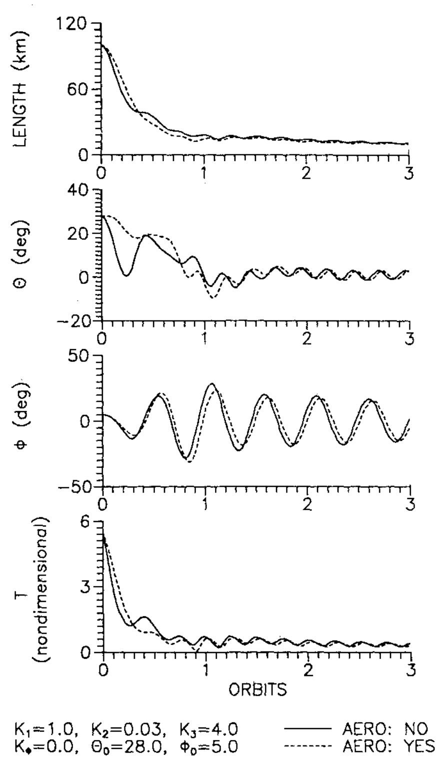

Fig. 3 Retrieval (no thrusters).

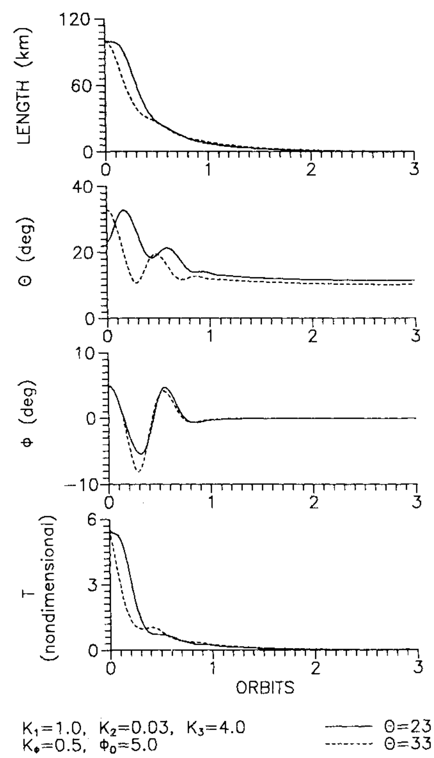

Fig. 5 Retrieval for different initial in-plane angles.

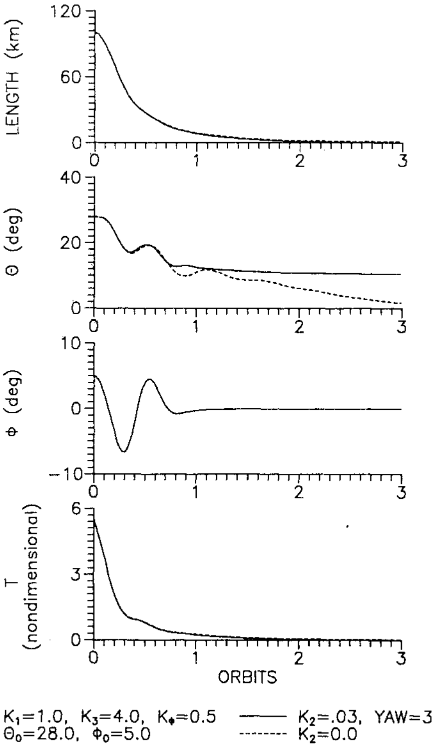

Fig. 6 Effects of pointing error and K_2 terms.

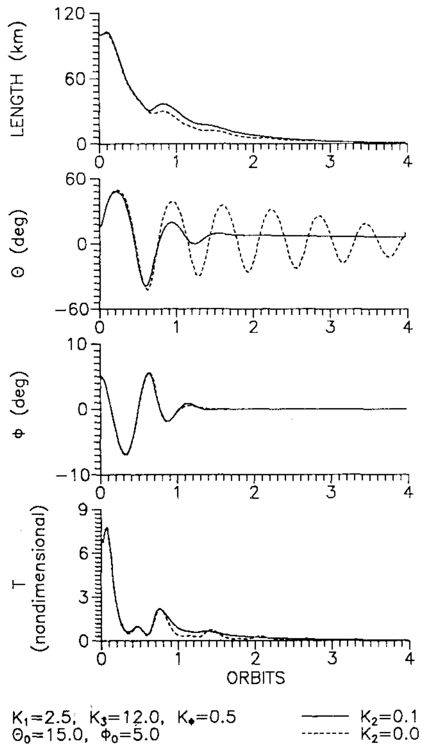

Fig. 7 Effect of K_2 terms with different gain values.

Retrieval

Some difficulty is encountered when the proposed tension control law is used for retrieval without out-of-plane thruster control. From Fig. 3, it can be seen that even without aerodynamic effects the retrieval rate is very slow during the end of the process due to large and slowly converging residual oscillation of the out-of-plane roll angle. However, if out-of-plane damping is included, convergence is faster and retrieval from 100 to 1 km can be accomplished within 2.57 orbits when aerodynamics is not considered and 2.41 orbits when aerodynamic effects are included (Fig. 4). After three orbits, the tether lengths are 0.553 and 0.447 km when aerodynamic effects are neglected and included, respectively. Also, in-plane and out-of-plane angular oscillations decay quickly within approximately 1.5 orbits. In Refs. 2 and 6, retrievals require approximately 4 orbits, even when librational damping is included in Ref. 2, and angular oscillations persist during the process. However, when damping is not considered, the tether lengths in Refs. 2 and 6 can be made to converge, whereas with the present control law, convergence is unacceptably slow without thruster augmentation.

Figure 5 shows a couple of typical retrieval responses for different initial in-plane angles. It was found that when aerodynamic effects are included the system may flip to the reversed equilibrium position if the initial pitch angle is far enough away from the initial steady-state hangoff angle of approximately 28 deg. Retrieval can be successfully accomplished if the initial in-plane angle is within 8–10 deg of the steady-state hangoff angle for the set of gains used in Fig. 5. It should be noted, however, that in most cases retrieval of the satellite will commence after a sufficiently long period of stationkeeping. Therefore, the initial pitch angle should be near the steady-state hangoff angle before retrieval, allowing the use of the proposed control law in normal situations.

Figure 6 shows the retrieval responses when the system is subjected to a pointing error of 3 deg in yaw and when the K_2 gain is set to zero. It can be seen that the pointing error has very little effect on the system by comparing with Fig. 4.

By setting K_2 to zero, the complex angular terms in the control law are eliminated and the control law reduces to a simple length error and velocity feedback law with thruster augmentation. Although it appears that the elimination of the K_2 terms have little effect on the overall system response due to the figure scaling, it was verified that even after 4 orbits the tether length is >1 km. Figure 7 shows the effect of eliminating the K_2 terms when a different set of gains are used. It can be seen that the terms multiplied by K_2 play a significant role in damping out the in-plane oscillations and allowing faster convergence of the tether length during the final phases of retrieval.

Conclusions

A nonlinear feedback control law for the deployment and retrieval of the tethered satellite system is developed using the Lyapunov approach for a dynamic model that includes tether mass and out-of-plane motion. This control law guarantees stability of the simplified tethered satellite system dynamic model developed herein when aerodynamic effects are neglected. Stability analyses were carried out to verify that no other equilibrium states other than the desired final state and the reversed state exist. Numerical simulations show that the control law, without thruster augmentation, will produce very fast deployment for applications where the aerodynamic effects may be neglected and successful, but oscillatory, deployment when aerodynamic effects are included. For retrieval, thruster augmentation in the out-of-plane direction is required for faster convergence during the terminal phase and for increased damping of the out-of-plane librations. Even when aerodynamic drag is considered, the thruster augmented control law will result in successful retrieval to 1 km within 2.5 orbits under normal situations (when the initial pitch angle is near the equilibrium hang-off angle). It was also found that for the gains used in the simulations the effects of constant thruster pointing error are insignificant. Verification of the control law in the presence of other effects such as tether flexibility and curvature have not been made and should be studied before actual implementation.

Acknowledgment

The support of the Texas Advanced Research and Technology Program (Projects 4193/1987 and 231/1989) is gratefully acknowledged.

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